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Annual Review

SCALING SPATIAL PREDICTABILITY: AN APPROACH TO MULTI-RESOLUTION MODELING

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Abstract—We have investigated the dependence of *spatial predictability*, a statistical measure of the reduction in uncertainty about one spatial variable that can be gained by knowledge of another, on the spatial resolution (R_s) of the variables. While increasing resolution provides more descriptive information about the patterns in data, it also increases the difficulty of accurately modeling those patterns. By examining the variation of spatial predictability with R_s in a number of case studies, we have proposed the existence of an "optimal" R_s for specific studies, which balances these two factors. We analyzed land-use data by resampling map data sets at several different spatial resolutions and measuring predictability at each. Spatial *auto-predictability* (P_a) is the reduction in uncertainty about the state of a cell in a map given knowledge of the state of adjacent cells in that map, and spatial *cross-predictability* (P_c) is the reduction in uncertainty about the state of a cell in a map given knowledge of the state of corresponding cells in other maps. The P_a is a measure of the internal pattern in the data, whereas P_c is a measure of the ability of some "model" to represent the transition from one map to another. We found a strong linear relationship between the log of P_a and the log of R_s (measured as the number of cells per square kilometer). While P_a generally increases with increasing R_s (because more information is being included), P_c generally falls or remains stable (because it is easier to model aggregate results than fine-grained ones). Thus, one can define an "optimal" R_s for a particular modeling problem that balances the benefit in terms of increasing data predictability (measured by P_a) as one increases resolution, with the cost of decreasing facility of modeling the temporal dynamics (measured by P_c).

Keywords—Resolution Predictability Spatial modeling Fractal

INTRODUCTION

We have investigated the dependence of *spatial predictability*, a statistical measure of the reduction in uncertainty about one spatial variable that can be gained by knowledge of another, on the spatial resolution of the variables. A spatial variable is defined as a "map," or a two-dimensional array of "cells," each of which exists in a state represented by an integer value. The terms "map" and "state distribution" will be used interchangeably. This analysis utilizes temporal series of land-use maps of Maryland and Florida, and the set of possible states (state-space) for each location is displayed in the legend of Figure 1. Each series of maps was generated by observing the actual landscape at several points in time and assigning a state to each cell based on this observation. The change in the state distribution over time is assumed to be driven by an implicit set of laws or "temporal dynamics," which we are attempting to mimic with a "model." We assume that a highly predictable temporal evolution implies the existence of relatively simple and easily modeled underlying temporal dynamics (and vice versa).

We can distinguish three types of resolution that are relevant to this study:

1. *Spatial resolution* (R_s). By spatial resolution we mean "grain size" or the size of the smallest unit of spatial measure, with increasing resolution corresponding to finer grain. In practice, it is measured as the number of cells in a map, assuming that all maps in the study cover the same geographic area.
2. *Temporal resolution* (R_t). Temporal resolution is a measure of the time-step between successive maps in a temporal dynamic series, with increasing temporal resolution representing a decreasing time-step.
3. *State-space resolution* (R_a). State-space resolution is a measure of the level of detail or "articulation" represented in the data that must be modeled. It is also a measure of the number of variables or degrees of freedom that will be necessary in constructing the model. In this study it is measured as the size of the state-space (i.e., the number of available states).

We hypothesize that an important determinant of the spatial predictability of spatiotemporal dynamics is the scale (resolution and extent) of the analysis. We can distinguish at least two ways that spatial resolution might affect spatial predictability. One is the increasing difficulty of building predictive models at increasing R_s. For example, it is easier to predict general climate patterns than it is to predict the exact geographic location and timing of rainstorms (the weather).

On the other hand, higher R_s allows more detail to be observed and internal patterns in the data to be seen that may not have been observed at cruder resolutions. One example

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is the warm core gyres that form in the Gulf Stream that were not observed until remote sensing images, including the proper thermal bands and of sufficiently fine resolution, were available. Another example is the quest by the military to obtain high-resolution satellite images to see the features (such as tanks and airplanes) of interest to them that would not appear on lower-resolution images.

Some phenomena are known to vary in a predictable way with resolution. For example, the analytical relationship between the measured length of a coastline and the resolution at which it is measured is a fundamental one behind the concept of fractals [1] and can be summarized in the following equation:

$$L = ks^{(1-D)} \tag{1}$$

where

L = the length of the coastline or other "fractal" boundary
s = the size of the fundamental unit of measure or the resolution of the measurement
k = a scaling constant
D = the fractal dimension.

Fractal analysis has been shown to play an important role in landscape analysis [2–7]. This convenient "scaling rule" has proved to be very useful in describing many kinds of complex boundaries and behaviors [5,8–11]. We hypothesized that this same kind of relationship might exist between resolution and predictability (and possibly other measures as well) and might be useful for developing scaling rules for choosing the appropriate resolution when designing a model. We tested this hypothesis by calculating two forms of predictability for a number of landscapes at a number of different spatial resolutions.

MEASUREMENT OF PREDICTABILITY

Colwell [12] applied information theoretic concepts to the problem of estimating the degree of predictability of periodic phenomena. The method is similar to autocorrelation analysis except that it is applicable to both interval and categorical data and may thus be more appropriate, for example, for comparing patterns of land cover. Predictability in this context refers to the reduction in uncertainty about one variable that can be gained by knowledge of another. For example, if the seasonal rainfall pattern in an area is predictable (e.g., there is always a severe dry summer), then knowing the time of year provides information about rainfall (if it is summer, it must be dry). If there is no relationship between rainfall and season, time of year tells us little and the rainfall is relatively unpredictable from a knowledge of time of year.

These techniques can also be applied to spatial data [13]. In this application, one is interested in the degree to which the uncertainty about the category of a particular cell is reduced from knowledge of other aspects of the same map, or from knowledge of aspects of other related maps. Several aspects of a map might be used as predictors. We discuss two implementations based on (a) the state of adjacent cells in the same map ("auto-predictability" or "spatial-adjacency predictability," P_a), and (b) the state of corresponding cells in other related maps ("spatial cross-predictability," P_c). Other combinations of these two and higher-level analyses (i.e., adjacent cell pairs, triplets, etc., or multiple cross-comparisons) are also possible and useful for various purposes [13].

The method in general can determine if there are regularities in a spatial data set, ranked on a scale from 0 (totally unpredictable) to 1 (totally predictable), and the answer can be interpreted as the degree of departure of the map or comparison between maps from a random (totally unpredictable) pattern.

To estimate spatial predictability, one first assembles a contingency matrix with states of the cells along the left axis, and corresponding states of other cells along the top. For auto-predictability the categories in a map are listed on the left and along the top of a matrix. The numbers in the matrices represent the frequency of occurrence in the mapped data of the category (or category pair, triplet, etc., for higher-level analysis) listed along the top of the matrix lying adjacent to the category listed along the left. This yields information about how predictable the patterns of adjacency are in the sample map data.

The contingency matrix can be any set of meaningful spatial relationships in the data. For example, another way of setting up the matrix is to define the spatial predictability of one map given another map. For example, we might want to know the spatial predictability of a landscape in one year given information in some previous year(s), or we might want to know the spatial predictability of a real landscape compared to a landscape model's output. We call this the "cross-predictability" because it provides information on the predictability of a given cell's category given knowledge of the category of the corresponding cell in another map.

Following Colwell [12], we define N_{ij} to be the elements in the contingency matrix (i.e., the number of times in the data that a cell of category i was adjacent to one of category j for auto-predictability analysis). Define X_j as the column totals, Y_i as the row totals, and Z as the grand total, or

$$X_j = \sum_{i=1}^{s} N_{ij} \tag{2}$$

$$Y_i = \sum_{j=1}^{t} N_{ij} \tag{3}$$

where s (t) is the total number of rows (columns) in the contingency matrix, and

$$Z = \sum_i \sum_j N_{ij} = \sum_j X_j = \sum_i Y_i. \tag{4}$$

Then the uncertainty with respect to X is

$$H(X) = -\sum_{j=1}^{t} \frac{X_j}{Z} \ln \frac{X_j}{Z} \tag{5}$$

and the uncertainty with respect to Y is

$$H(Y) = -\sum_{i=1}^{s} \frac{Y_i}{Z} \log \frac{Y_i}{Z} \tag{6}$$

and the uncertainty with respect to the interaction of X and Y is

$$H(XY) = -\sum_i \sum_j \frac{N_{ij}}{Z} \log \frac{N_{ij}}{Z}. \quad (7)$$

Then define the conditional uncertainty with regard to Y with X given as

$$H_x(Y) = H(XY) - H(X). \quad (8)$$

Finally, define a measure of predictability (P) with the range (0,1) as

$$P = 1 - \frac{H_x(Y)}{\log s} = 1 - \frac{H(XY) - H(X)}{\log s}, \quad (9)$$

where s is the total number of rows (categories) in the contingency matrix.

This measure gives an index scaled on the range from 0 (unpredictable or maximum uncertainty) to 1 (totally predictable or minimum uncertainty). Predictability will be minimal when all the elements in the contingency matrix (N_{ij}) are equiprobable (i.e., when all entries are the same), and will be maximized when only one entry in each column is nonzero. Most real spatial data will fall between these extremes.

STUDY AREAS

We applied these indices of spatial predictability to land-use data sets from the Kissimmee/Everglades Basin, Florida, and the state of Maryland. Both of these data sets contained three distinct years of data over which significant changes in land-use patterns had occurred.

Kissimmee/Everglades basin, Florida

The Kissimmee/Everglades drainage basin in south Florida represents one of the most rapidly changing and intensively modified landscapes in the country. It consists of some 18,700 square miles (48,500 km^2) of land and water, covering about one-fourth of the state of Florida. A set of three land-use maps with 26 land-use categories was prepared for the years 1900, 1953, and 1973 in order to analyze the dramatic changes that had occurred in the region during this interval [14,15]. In 1900, when much of the United States had already been developed into farmland and cities, the Kissimmee/Everglades basin remained much as it had been for centuries. By 1953, the expanding U.S. middle-class and postwar economic boom had led to significant urban and agricultural development. This development accelerated into the 1970s and 1980s.

For the spatial predictability analysis we used versions of the land-use maps that had been manually digitized into 128-acre (0.52 km^2) rectangular cells, each 625 by 833 m [15]. This produced an array with overall dimensions of 576 rows by 400 columns (230,400 total cells) of which about 93,000 were inside the study area. Each cell was assigned one of 26 land-use categories ranging from natural to agricultural to urban systems [15].

State of Maryland

The Maryland Department of State Planning has compiled digitized land-use data (using 80-acre cells) for three different years (1973, 1981, and 1985) for the entire state, using 20 different land-use categories. While there were significant shifts in land use in Maryland, land use had changed much less dramatically in this area over the 12-year interval between 1973 and 1985 than it had in south Florida during the 1900 to 1973 interval. Major changes involved reforestation of agricultural areas and significant increases in urban land uses, especially in the Washington/Baltimore corridor. For the state of Maryland we used a rectangular array of 345 rows by 640 columns or a total of 220,800 cells, of which approximately 68,800 were within the boundaries of the state.

SOFTWARE AND ALGORITHMS

Land-use data from the two study sites were imported into Map II®, a simple and easy-to-use raster-based geographic information system (GIS) package for the Macintosh, to prepare data files for calculations using high-speed parallel transputers, as discussed below. Any GIS system capable of producing raster output would be suitable, however.

Decreasing the spatial resolution (increasing the grain) of a spatial data set involves the repetitive resampling of a specified number of smaller cells into larger cells. Analytically, this is accomplished by moving a resampling matrix (whose size is the number of rows and columns of the original data needed to make a single cell in the new raster) through the original raster. The cell values falling within the resampling matrix are tabulated and used to determine the value of the appropriate larger cell in the new, coarser resolution raster. We experimented with several methods of resampling or aggregating the spatial data.

The first method, which we call *proportional aggregation*, assigns the cell values in the coarser-grain raster according to the most dominant category found within the resampling matrix. A second method, termed *random aggregation*, assigns new categories by randomly choosing from the categories found within the resampling matrix. The major difference between the two methods is that rare categories are more likely to be preserved when the data are resampled with the random aggregation scheme. While the choice of aggregation scheme can be significant in many spatial analyses, we found that the aggregation scheme made little difference to the results of our particular experiments. We settled on a version of the random aggregation scheme that is both simple and suits our immediate needs. In this version aggregation takes place in steps. In each step the original map is aggregated using a 2×2 resampling matrix, yielding an aggregated map with one-fourth the number of cells of the original. In each 2×2 resampling matrix we choose the category of the northwest cell as the category for the cell in the aggregate map. This process was repeated on the new aggregate map to yield a series of maps each with one-fourth the total number of cells of the one preceding it in the series. Each map in this series has one-fourth the resolution of the preceding map. For the Florida maps the resulting resolutions (in cells/km^2) were 1.333 (original), 0.333, 0.083, 0.021, and 0.005. For the

Maryland maps the resulting resolutions (in cells/km^2) were 2.743 (original), 0.686, 0.171, 0.043, and 0.011.

We developed algorithms in a parallel version of the C programming language to calculate auto- and cross-predictability for mapped data on Inmos Transputers (a form of RISC-based parallel processor) on a Macintosh [16]. Each map was parceled into four equal-sized submaps, and the four submaps were processed in parallel, each by a separate transputer. Transputers are extremely fast for this sort of calculation. For example, for the south Florida data (a 576 × 400 array) calculation of auto-predictability at five levels of resolution and printing results to a text file took approximately 2.4 seconds using a Macintosh IIci with four transputers.*

AUTO-PREDICTABILITY EXPERIMENTS

We calculated P_a for both study areas for all three years and at five different spatial resolutions (R_s), holding state-space resolution R_a constant. We started with the maximum R_s of the data and gradually degraded it by aggregating cells. In each step we halved R_s by aggregating 2 × 2 blocks of cells at the previous R_s into single cells as described above. Spatial resolution is frequently indicated as the length of a side of a cell, with higher or finer resolution corresponding to smaller cell sizes. For example, LANDSAT satellite data has 30-m resolution while SPOT satellite data has finer resolution at 18 m. In our plots we wanted higher resolution to correspond to higher (not lower) numbers, so we measured resolution as the number of cells per km^2. For example, 50-m cells would have a resolution of 400 cells/km^2, while 200-m cells would have a resolution of 25 cells/km^2. We fit the equation

$$P = kr^{(1-D_P)}, \tag{10}$$

where

P = the spatial predictability (P_a refers to auto-predictability; P_c refers to cross-predictability)
r = the resolution measured as the number of cells/km^2
k = a unitless scaling factor
D_P = the fractal predictability dimension (unitless),

by first transforming it into log-log form:

$$\ln P = \ln(k) + (1 - D_P)\ln(r) \tag{11}$$

and using standard linear regression analysis to solve for the parameters k and D_P.

Results are summarized in Table 1, which indicates the high R^2 for this relationship for both of the study sites.

CROSS-PREDICTABILITY EXPERIMENTS

We calculated P_c for both of the study areas by comparing maps from different years. We hypothesized the existence of an implicit spatiotemporal dynamics that generates the state distribution at time $t + 1$ given the state distribution at time t. The goal of the modeler is to make the implicit dynamics explicit (i.e., express it as a set of equations), and P_c can then be interpreted as a measure of how difficult this task will be. We were interested in studying the relationship between P_c and R_s. We measured P_c for five values of R_s and fit Equation 11 to the data. The results for the two sites evaluated at several time points are shown in Table 2.

Results of both the auto- and cross-predictability experiments for both sites are plotted together on a log–log scale in Figure 1. The strong linearity of the relationship for all cases is apparent, as is the fact that auto-predictability (P_a) increases with increasing spatial resolution (R_s) while cross-predictability (P_c) decreases slightly with increasing R_s, although with a smaller D_P. These results are consistent with our original hypotheses. Results for the Kissimmee/Everglades data are markedly different from those for the Maryland data. The auto-predictability of Maryland land use changed much less with R_s than the Kissimmee/Everglades land use. The slope of the regression line ($1 - D_{AP}$) for the Kissimmee/Everglades data was roughly three times that for

Table 1. Fractal auto-predictability dimension (given as $1 - D_{AP}$), scale constant (k), adjusted R^2, and degrees of freedom ($d.f.$) for auto-predictability (P_a) from regression of Equation 3 for both data sets

Site and year	k	$(1 - D_{AP})$	Adj. R^2	$d.f.$
Kissimmee/Everglades, Florida				
1900	0.6364	0.111	0.999**	4
1953	0.6383	0.085	0.988**	4
1973	0.6250	0.096	0.981**	4
all years	0.6332	0.097	0.958**	14
State of Maryland				
1973	0.5189	0.031	0.780*	4
1981	0.5046	0.034	0.780*	4
1985	0.4956	0.030	0.631*	4
all years	0.5434	0.031	0.720**	14

** Indicates significant at the 0.01 level; * indicates significant at the 0.05 level.

Table 2. Fractal cross-predictability dimension (stated as $1 - D_{CP}$), scale constant (k), adjusted R^2, and degrees of freedom ($d.f.$) from regression of Equation 3 for cross-predictability (P_c) for both data sets

Site and year	k	$(1 - D_{CP})$	Adj. R^2	$d.f.$
Kissimmee/Everglades, Florida				
1900/1953	0.5764	−0.011	0.943**	4
1953/1973	0.4936	−0.017	0.778*	4
State of Maryland				
1973/1981	1.0790	−0.006	0.805*	4
1981/1985	0.9296	−0.004	0.777*	4

** Indicates significant at the 0.01 level; * indicates significant at the 0.05 level.

*The algorithms also work on serial machines, only slower. Each transputer is approximately the speed of a SUN Sparc station so the transputer time is about four times the speed one would expect on a Sparc station. Contact Tom Maxwell for more information about obtaining the algorithms or using transputers for spatial analysis.

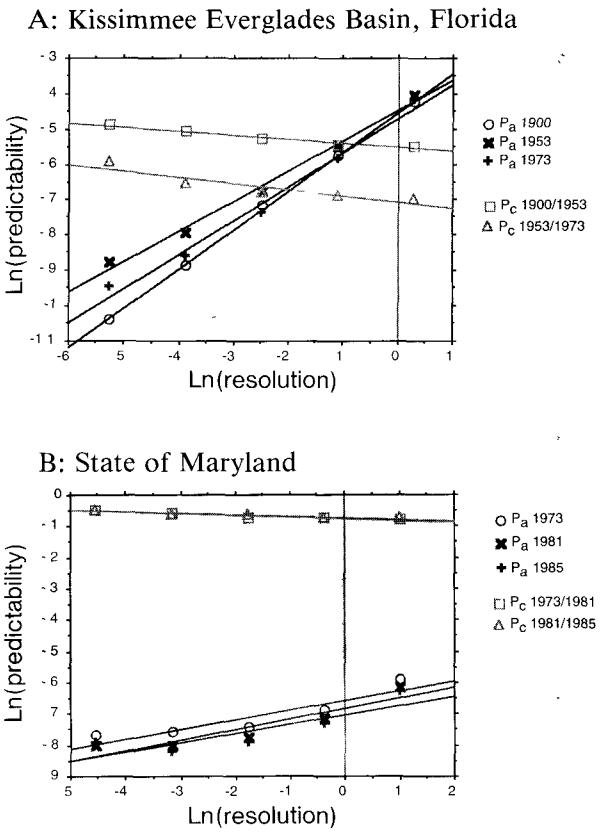

Fig. 1. Natural log of resolution vs. natural log of predictability for **(A)** the Kissimmee/Everglades, Florida, and **(B)** the state of Maryland land-use data. Plot shows both auto-predictability (P_a) indicating internal pattern in the data for three different years, and cross-predictability (P_c) indicating pattern matching between null models of prior land-use maps and the particular map. The resolutions used (in cells/km^2) were: Florida, 0.005, 0.021, 0.083, 0.333, 1.333; Maryland, 0.011, 0.043, 0.171, 0.686, 2.743.

the Maryland data. Auto-predictability varied from about 0.65 to 0.35 over the range of resolutions used for the Florida data, but only from about 0.55 to 0.45 for the Maryland data. The Kissimmee/Everglades data were also more predictable at the highest resolutions than were the Maryland data.

The cross-predictability results also differ markedly between the Florida and Maryland data. The slope of the regression line ($1 - D_{CP}$) was about three times higher for the Florida data than the Maryland data. The difference in P_c and P_a results between the Maryland and Florida data is largely due to the fact that the Maryland temporal dynamics are sampled at a higher rate, yielding data at a higher temporal resolution (R_t). We see that increasing R_t results in increasing P_c and P_a, owing to the fact that increased temporal resolution will result in decreased variation in state-space and thus increasing predictability. We conjecture that increasing state-space resolution R_a would result in decreasing P_c and P_a, due to the fact that increasing the number of possible states makes prediction more difficult.

DISCUSSION AND CONCLUSIONS

We can draw several conclusions from our analysis. First, P_a and P_c belong to a class of measures (inspired by "fractal dimension") that vary in a regular way as resolution changes. This allows an index to be calculated that permits easy conversion of measurements of P taken at one resolution to other resolutions (for example, resolutions higher than those for which we have data). This measure, together with other measures (including those documented and those yet to be discovered) that also exhibit this kind of self-similarity, may be useful in developing a generalized theory of scaling.

Second, P_a generally increases with increasing spatial resolution (R_s). This relationship represents the "benefit" in terms of information gained about the pattern as R_s is increased.

Third, P_c generally falls with increasing R_s. This relationship represents the "cost" of decreased model predictability as R_s is increased.

Fourth, combining the second and third conclusions leads to some hypotheses about determining an "optimal" spatial resolution (R_s) for specific studies. At very low R_s it is easier to build predictive models, but they have little useful detail. At high R_s much useful detail is retained, but models are less able to predict it. An optimal R_s for scientific analysis may occur where these trends intersect (Fig. 2)—where one is balancing the costs and benefits incurred with increasing R_s. These results are consistent with empirical data from a survey of over 85 models of freshwater wetlands [17].

Fifth, we suspect that the optimal resolution may vary depending on the class of temporal state-space dynamics that is observed in the map series (and subsequently modeled), and possibly for each particular set of modeling objectives. We also suspect that P_c and its associated D_{CP} will change with changing technology and modeling skills. We are cur-

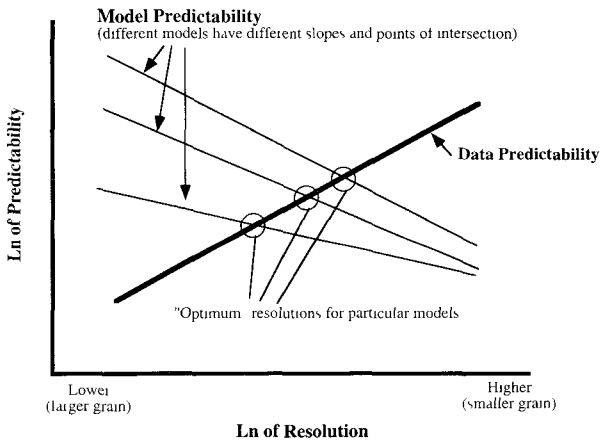

Fig. 2. Hypothetical relationship between resolution and predictability of data and models. Data predictability is the degree to which the uncertainty about the state of landscape cells is reduced by knowledge of the state of adjacent cells in the same map. Model predictability is the degree to which the uncertainty about the state of cells is reduced by knowledge of the corresponding state of cells in output maps from various models of the system.

rently pursuing research aimed at addressing these questions by applying process-based spatial models at several different resolutions.

Finally, other indices may exist that exhibit similar behavior for resolution measures other than R_s (e.g., temporal [R_t] and state-space [R_a]) and may shed some interesting light on "chaotic" behavior in systems. When looking across resolutions, chaos may be the low level of model predictability that occurs as a natural consequence of high resolution. Lowering model resolution can increase model predictability by averaging out some of the chaotic behavior at the expense of losing detail about the phenomenon. For example, Sugihara and May [11] found chaotic dynamics for measles epidemics at the level of individual cities, but more predictable periodic dynamics for entire nations. The idea is not to maximize the resolution of analysis so as to "discover" this "unpredictable" chaotic behavior, nor is it to maximize predictability by ignoring details. Rather, the aim is to choose the resolution that maximizes the *effectiveness* of the model in balancing the conflicting trends of data and model predictability with changing resolution.

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